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Optimal Monetary and Fiscal Policy Rules, Welfare Gains, and Exogenous Shocks in an Economy with Default Risk *

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Abstract

The European debt crisis motivated us to develop policy prescriptions to address the issue from the perspective of implementing optimal monetary and fiscal policy. We develop a class of dynamic stochastic general equilibrium models with nominal rigidities and introduce default risk to the model. We find that if productivity changes are observed, policy authorities should be aware of default risk, although being aware of such risk is not essential following government expenditure changes. Welfare gains from awareness of default risk are nonnegligible if productivity changes, although welfare gains from awareness of default risk are minimal following government expenditure changes. In other words, in response to a change in productivity, stabilizing inflation should be modest and consider suppressing default; however, inflation can be stabilized aggressively without taking into account the suppressing of default in response to a change in government expenditure.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level

JEL Classification: E52; E60

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1 Introduction

> The Governing Council decided to lower the key ECB interest rates by 25 basis points, following the 25 basis point decrease on 3 November 2011. Inflation is likely to stay above 2% for several months to come, but it will decline to below 2% during 2012. The intensified financial market tensions are continuing to dampen economic activity in the Euro area and the outlook remains subject to high uncertainty and substantial downside risk.

The above is an extract of a speech by Vítor Constâncio, vice-president of the European Central Bank (ECB) on Dec. 8, 2011.[1] At the time, the eurozone was hit by a sovereign debt crisis: the Greek 10-year credit default swap premium reached USD 20,404 and the Harmonised Consumer Price Index (HCPI) inflation was 2.8% in April 2012. This speech describes the difficulty of implementing monetary policy amid a sovereign debt crisis, implying that the ECB may have prioritizes preventing the spread of the crisis over stabilizing inflation by lowering the policy rates to provide liquidity and promote debt deflation at that time.

In this paper, we attempt to address the difficulties faced by the ECB during the crisis period and provide important prescriptions for formulating an optimal monetary and fiscal policy in an economy with default risk from the viewpoint of minimizing welfare costs as follows: 1) policy authorities should not suppress inflation aggressively if the cause of the default crisis is rooted in a decrease in productivity. In response to a decrease in a productivity, if the focus is solely on stabilizing inflation, the fiscal authority decreases the tax rate to stabilize inflation by boosting output. Although inflation is stabilized, default becomes inevitable because a decrease in tax rate harms government solvency. If the aim is to stabilize inflation and suppress default, that is, to implement an optimal monetary and fiscal policy, in response to a decrease in productivity, although the fiscal authority decreases tax rate, the decrease is less, partially suppressing default and stabilizing inflation. However, this is optimal and we advocate for this policy prescription, which is not novel but is consistent with our intuition. However, we have another prescription: 2) when an increase in government expenditure gives rise to default risk, policy authorities should stabilize inflation, similar to a situation in which there is no default risk. If the focus is solely on stabilizing inflation, the fiscal authority increases the tax rate to counteract the pressure boosting output resulting from an increase in government expenditure. This increase in tax rate improves government solvency. Thus, stabilizing inflation causes suppressing default, achieving a result similar to that under optimal policy. The latter prescription may be welcomed by policy authorities.

In connection with previous prescriptions, we have: 3) welfare gains are negligible from implementing an optimal monetary and fiscal policy with awareness of default risk when there is an increase in government expenditure; while 4) welfare gains are nonnegligible from implementing an optimal monetary and fiscal policy with awareness of default risk when there is a decrease in productivity.

To derive the above prescriptions, we analyze optimal monetary and fiscal policy in an economy with default risk. First, we develop a class of dynamic stochastic general equilibrium (DSGE) models in which default risk is introduced following Okano and Inagaki[17] who replicated Uribe's[19] fiscal theory of sovereign risk (FTSR) in the DSGE. Although Calvo pricing is assumed, the steady state is distorted because tax is levied on the output. There exist safe assets issued by households

[1] See ECB[10].

and risky assets, namely government debt. The FTSR is applicable, derived from the government budget constraint that is iterated forward and an appropriate transversality condition.[2] Let us suppose a decrease in the net present value of the fiscal surplus. According to the fiscal theory of price level (FTPL), this increases the price level, while under the FTSR, there exists the possibility of causing an increase in the default rate instead of an increase in the price level. Next, we derive the second-order approximated utility function, which includes not only a quadratic inflation term but also a quadratic premium difference between the (virtual) government debt yield and its coupon rate term, which we call the premium difference, implying that the cost of default risk is summarized as the premium difference.

As well as explaining the reason for a quadratic term for the premium difference in the second-order approximated utility function, we now expand our introduction to our model. Our setting includes both safe and risky assets, namely government debt. If households purchase government debt, their optimal consumption schedule aligns the intertemporal marginal rate of substitution to the inverse of the gross expected rate of return to holding government debt, which comprises the government debt yield and the expected default rate. Thus, households have to adjust their balance of government debt appropriately. By adjusting the balance, which affects the inverse of the gross expected rate of return on holding government debt through changes in the government debt yield, the optimal consumption schedule is achieved. If the government debt coupon rate precisely corresponds to the government debt yield, such an adjustment is not needed. However, that is not necessarily common in an actual economy.[3] Thus, adjusting the balance of government debt, namely portfolio rebalancing, is essential. The premium difference is a function of the expected default rate, and the appearance of the quadratic term on the period welfare cost function implies that the premium difference is the cost of default risk. In other words, default risk generates a cost, forcing households to rebalance their portfolio.

In this paper, we analyze both *Exact* and *False* policies. Under the *exact* policy, the *exact* welfare cost function is minimized by policy authorities, the central bank, and the government while under the *false* policy, they minimize the *false* welfare cost function. The *exact* welfare cost function is derived exactly, assuming default risk, while the *false* welfare cost function is derived without assuming default. The two differences between the *exact* and the *false* welfare cost functions are the target level of output and the existence of a quadratic term for the premium difference. The difference in the target level of output between the *exact* and the *false* welfare cost functions depends on the interest rate spread in the steady state, which decides the steady-state value of the default rate. If the interest rate spread is zero (the steady-state value of the default rate is zero simultaneously), the target level of output in both welfare cost functions is the same. Similarly, the existence of the quadratic term of the premium difference in the *exact* welfare cost function depends on the interest rate spread in the steady state, while this term does not definitively appear on the *false* welfare cost function. If the interest rate spread in the steady state is zero, the premium difference becomes zero and its quadratic term spontaneously disappears from the *exact* welfare cost function. These two differences depend on the interest rate spread in the steady state, which, if zero, results in the *exact* welfare cost function precisely corresponding to

[2]The FTSR is based on the fiscal theory of price level (FTPL) advocated by Cochrane[6], Leeper[14] and Woodford[20] and the net present value of the sum of fiscal surplus decides not only price level and inflation but also the default rate.

[3]The government debt yield is consistent with the coupon rate on benchmark 10-year government bonds in Italy, Spain, Germany, and the United States. However, in Portugal, Ireland, and Greece, the yield is not consistent with the coupon rate on the benchmark 10-year government bond. See Okano and InagakiOkanoInagaki17 for details.

the *false* welfare cost function. Because the interest rate spread in the steady state decides the steady-state value of the default rate, the default risk can be said to affect the period welfare cost function, that is, the policy target. If default risk exists, authorities have to focus on the target level of output and consider minimizing the premium difference.

We resort to numerical analysis with plausible parameterization and compare the results under the *exact* and *false* policies. The impulse response functions (IRFs) imply that policy authorities should not suppress inflation aggressively if the causation of the default crisis is rooted in a decrease in productivity. However, policy authorities should stabilize inflation, as they would in a scenario with no default risk if an increase in the government expenditure results in default risk. Furthermore, we also calculate "optimal" monetary and fiscal policy rules, by choosing the coefficients on simple rules that belong to classes of monetary and fiscal policy rules, replicating the welfare costs brought about by an optimal monetary and fiscal policy. Interestingly, no differences exist in our fiscal policy rule between *exact* and *false* policies regarding an increase in government expenditure. This result is consistent with our second policy prescription and implies that even if default risk exists, inflation should be stabilized aggressively as long as default risk results from an increase in government expenditure. However, when productivity decreases, the rules are quite different. This result is consistent with our first policy prescription and implies that policy authorities should not suppress inflation aggressively if default risk results from an increase in productivity.

Finally, we calculate welfare cost by adopting the *exact* policy rule, finding that the welfare gains from implementing the *exact* policy are 64.4%. These gains are remarkable. Furthermore, concerning changes in productivity, the gains are 49.7%. However, when only government expenditure hits an economy with default risk, the gains are a negligible 1.0%.

We now discuss our analysis with previous work deriving policy implications in an economy with default risk. Corsetti and Dedola[7] developed a model for a sovereign debt crisis driven by either self-fulfilling expectations or weak fundamentals, analyzing the mechanism through which either conventional or unconventional monetary policy can preclude the former. Their finding that swapping government debt for monetary liabilities can prevent self-fulfilling debt crises is one of several unconventional monetary policies. Similar to our analysis, Bacchetta, Perazzi, and Wincoop[1] developed a class of DSGE models and analyzed conventional and unconventional monetary policies. They found that the central bank cannot credibly avoid a self-fulfilling debt crisis. Okano and Hamano[16] and Okano and Inagaki[17] analyzed stabilizing inflation and suppressing default trade-offs, finding that these do not necessarily exist. Our analysis differs from this earlier body of work in several ways. Although Corsetti and Dedola[7] and Bacchetta, Perazzi, and Wincoop[1] analyzed monetary policy, they did not consider fiscal policy or its use as a stabilization or welfare cost-minimization tool. Thus, our purposes are not identical because while we propose monetary and fiscal policies, these related studies proposed monetary policy only to suppress default risk.[4] Okano and Hamano[16] and Okano and Inagaki[17] fail to derive implications for welfare costs, which is a focus in our work.

The remainder of the paper is structured as follows. Section 2 develops the model, Section 3 derives the welfare cost function and solves the linear quadratic (LQ) problem, Section 4 is devoted to numerical analysis, and Section 5 calculates monetary and fiscal policy rules and analyzes welfare costs. Section 6 concludes the paper. Appendices provide some technical information.

[4]Furthermore, these studies do not focus on fiscal policy (their models are unsuitable for analyzing fiscal policy regardless), whereas our model can analyze and evaluate the effect of fiscal policy.

2 The Model

Following Okano and Inagaki[17], we introduce firms into Uribe's[19] FTSR and following Gali and Monacelli[13], we develop a class of DSGE models with nominal rigidities although we assume a closed economy.[5] The default mechanism is quite similar to Uribe[19]. We follow Benigno[2] (an earlier working paper version of Benigno[3]) to clarify the households' choice of risky assets. The household i on the interval $i \in [0,1]$ supplies labor and owns firms. We adopt Calvo pricing and assume that a tax is levied on output, distorting outcomes. Thus, monopolistic power remains and the steady state is distorted.

2.1 Government

We assume that the total government expenditure is given exogenously in each period by $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ where ε denotes the elasticity of substitution among goods. The flow government budget constraint is represented by:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - \int_0^1 P_t(i) \left[\tau_t Y_t(i) - G_t(i)\right] di,$$

where $R_t^G \equiv R_t \Gamma(-sp_t)$ is the government debt coupon rate, $R_t \equiv 1 + r_t$ is the gross (risk-free) nominal interest rate, r_t is the net interest rate, B_t^n is the nominal government debt, δ_t is the default rate, $sp_t \equiv \frac{SP_t}{SP} - 1$ is the percentage deviation of the (real) fiscal surplus from its steady-state value, $SP_t \equiv \tau_t Y_t - G_t$ is the (real) fiscal surplus, and τ_t is the tax rate. We define $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ where Y_t denotes (aggregated) output. Because government expenditure is given exogenously, fiscal policy involves choosing the mix between taxes and the one-period nominal debt with default risk to finance the exogenous process of government expenditure.

Here, we discuss the government debt coupon rate $R_t^G \equiv R_t \Gamma(-sp_t)$, where $\Gamma'(-sp_t) > 0$ by assumption. Our assumption implies that the government decides the government debt coupon rate depending on its fiscal situation, such that if the fiscal situation worsens, the coupon rate is increased. The government debt coupon rate R_t^G is not the government debt yield, which is fully endogenized. In our setting, the government debt yield is decided by households' intertemporal optimal condition, namely the Euler equation. Thus, the government debt yield is decided endogenously, although the government debt coupon rate depends on our assumption.

As mentioned, the function $\Gamma(-sp_t)$ was suggested by Benigno[2], who developed a two-country model with imperfect financial integration, although the details differ from Benigno[2]. Benigno[2] assumed that households in the home country face a burden in international financial markets. As borrowers, the households will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration less than the foreign interest rate. Following his setting, Benigno[2] assumed $\Gamma'(\cdot) < 0$, which implies that the higher the foreign country's government debt, the lower the remuneration for holding the foreign country's government debt.[6] In contrast, our setting implies that the lower the fiscal surplus, the lower the remuneration for holding government debt owing to default, which in turn harms capital and makes households reluctant to hold government

[5] Following Ferrero[11], we introduce government into Gali and Monacelli[13]. In other words, this model is a closed economy version of Okano[15].

[6] Benigno[2] observed that this function, contingent only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

debt. The government has to pay additional remuneration for holding government debt, which provides households with motivation for doing so. Thus, we assume that $\Gamma'(\cdot) > 0$. That is, the lower the fiscal surplus, the higher the interest rate multiplier, and vice versa.

Another assumption that differs from Benigno[2] is that $\Gamma(\cdot)$ is a function of the fiscal surplus, which Benigno[2] assumed is a function of current government debt with an interest payment; that is, $R_t B_t$. Our setting for $\Gamma(\cdot)$ follows Corsetti, Kuester, Meier, and Mueller[8] indirectly. Corsetti, Kuester, Meier, and Mueller[8] assumed that the higher the fiscal deficit, the greater the probability of default, and vice versa. If it is given that the higher the probability of default, the higher the government debt coupon rate, our assumption that $\Gamma(\cdot)$ is a decreasing function of the fiscal surplus is consistent with their analysis because the assumption implies that the higher the fiscal surplus, the higher the government debt coupon rate. That is, if it is given that the higher the probability of default, the higher the government debt coupon rate, we can indirectly assume that the lower the fiscal surplus, the higher the default rate, which is similar to Corsetti, Kuester, Meier and Mueller[8].[7]

It is noteworthy that Schabert[18] argued that the equilibrium allocation cannot be determined if the central bank sets the interest rate in a conventional way. However, if money supply is controlled, the equilibrium allocation can uniquely be determined under Uribe's[19] FTSR. We adopt Uribe's[19] FTSR and we do not introduce money into our model. However, this does not definitely imply that the equilibrium allocation cannot be determined because we follow Benigno[2]; thus, the households' choice of risky assets is determined uniquely, allowing the equilibrium allocation to be uniquely determined.

The log-linearized definition of the fiscal surplus is given by:

$$sp_t = \varsigma_\tau \hat{\tau}_t + \varsigma_\tau y_t - \frac{\varsigma_\tau \sigma_G}{\tau} g_t, \tag{1}$$

where $\varsigma_\tau \equiv \frac{\tau}{\frac{SP}{Y}}$ denotes the tax revenue elasticity, $\sigma_G \equiv \frac{G}{Y}$ denotes the steady-state share of the government expenditure to output, and $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value. We refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ as the tax gap.

By solving cost-minimization problems, the optimal allocation of generic goods is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ and $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$, and the previous flow government budget constraint can be rewritten as:

$$B_t^n = R_{t-1}^G (1-\delta_t) B_{t-1}^n - P_t SP_t,$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}} \tag{2}$$

denotes the price level. Dividing both sides of the equality by P_t yields:

$$B_t = R_{t-1}^G (1-\delta_t) B_{t-1} \Pi_t^{-1} - SP_t, \tag{3}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being the gross inflation rate. The first term on the right-hand side (RHS) corresponds to the amount of redemption with nominal interest payment and reveals that the

[7] Our setting on $\Gamma(\cdot)$ follows Okano and Inagaki[17] who analyzed whether a fiscal deficit or government debt with interest payment increases the interest rate multiplier $\Gamma(\cdot)$ using Greek data. These data imply that the fiscal deficit but not government debt with interest payment increases $\Gamma(\cdot)$.

lower the past fiscal surplus, the higher the interest payments, and the higher the default rate, the lower the redemption, and vice versa.

Log-linearizing Eq. (3) yields:

$$b_t = \frac{\tau}{\beta\left(\tau+\phi\varsigma_\tau\sigma_B\right)}\hat{r}^G_{t-1} - \frac{\phi\varsigma_\tau\sigma_B}{\beta\left(\tau+\phi\varsigma_\tau\sigma_B\right)}\hat{\delta}_t + \frac{\tau}{\beta\left(\tau+\phi\varsigma_\tau\sigma_B\right)}b_{t-1} - \frac{\tau}{\beta\left(\tau+\phi\varsigma_\tau\sigma_B\right)}\pi_t - \frac{\tau}{\varsigma_\tau\sigma_B}sp_t, \quad (4)$$

with $\hat{r}^G_t \equiv \frac{dR^G_t}{R^G}$ where $\hat{\delta}_t \equiv \frac{d\delta_t}{\delta}$ denotes the default gap.

Here, we demonstrate that the log-linearized definition of the government debt coupon rate is given by:

$$\hat{r}^G_t = \hat{r}_t - \phi sp_t. \quad (5)$$

2.2 Households

2.2.1 The First-Order Necessary Conditions (FONCs) for Households

A representative household's preference is given by:

$$\mathcal{U} \equiv \mathrm{E}_0\left(\sum_{t=0}^{\infty}\beta^t U_t\right), \quad (6)$$

where $U_t \equiv \ln C_t - \frac{1}{1+\psi}N_t^{1+\psi}$ denotes the period utility, E_t is the expectation conditional on the information set at period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index, $N_t \equiv \int_0^1 N_t(i)\,dh$ represents the hours of labor, and ψ represents the inverse of the elasticity of labor supply.

The consumption index of the continuum of differentiated goods is defined as follows:

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (7)$$

where $\varepsilon > 1$ is the elasticity of substitution across goods.

The maximization of Eq. (6) is subject to a sequence of intertemporal budget constraints of the form:

$$R_{t-1}D^n_{t-1} + R^G_{t-1}B^n_{t-1}(1-\delta_t) + W_tN_t + PR_t \geq \int_0^1 P_t(i)\,C_t(i)\,di + D^n_t + B^n_t, \quad (8)$$

where D^n_t denotes the safe assets issued by households, W_t is the nominal wage, and PR_t denotes profits from the ownership of firms. Furthermore, we define V as the steady-state value of any variables V_t and v_t as the percentage deviation of V_t from its steady-state value.

By solving cost-minimization problems for households, we arrive at the optimal allocation of expenditures as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}C_t. \quad (9)$$

After accounting for Eq. (9), the intertemporal budget constraint can be rewritten as:

$$R_{t-1}D^n_{t-1} + R^G_{t-1}B^n_{t-1}(1-\delta_t) + W_tN_t + PR_t \geq P_tC_t + D^n_t + B^n_t.$$

The remaining optimality conditions for the household's problem are given by:

$$\beta\mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\right) = \frac{1}{R_t}, \quad (10)$$

which is the intertemporal optimality condition, namely the Euler equation, and

$$C_t N_t^{\psi} = \frac{W_t}{P_t}, \tag{11}$$

which is the standard intratemporal optimality condition.

Another intertemporal optimality condition depicting households' motivation to hold government debt with default risk exists. It is obtained by differentiating the Lagrangian by government nominal debt and is given by:

$$\lambda_t = \beta R_t^H \mathrm{E}_t \left[\lambda_{t+1}\left(1-\delta_{t+1}\right)\right], \tag{12}$$

with $\lambda_t = \left(P_t C_t\right)^{-1}$ where $R_t^H \equiv R_t \left\{\Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right) SP^{-1}\right\}$ is one of the earning rates or marginal revenue of holding government debt that can be interpreted as the (gross) government debt yield (excluding the default risk) requested by households. The definition of government debt yield R_t^H implies that household demand for government debt, which establishes the government debt yield discounting default risk $R_t^H \mathrm{E}_t\left(1-\delta_{t+1}\right)$, corresponds to the inverse of the marginal rate of consumption, namely R_t, as long as the additional interest payment for holding government debt is not sufficient to realize the optimal consumption schedule from holding government debt. Hereafter, we label R_t^H the government debt yield.

In fact, by combining log-linearizing Eqs. (10) and (12), we arrive at:

$$\hat{r}_t = \hat{r}_t^H - \delta \mathrm{E}_t\left(\hat{\delta}_{t+1}\right), \tag{13}$$

with $\hat{r}_t \equiv \frac{dR_t}{R}$ and $\hat{r}_t^H \equiv \frac{dR_t^H}{R^H}$ being the nominal interest gap and the government debt yield gap, respectively. Eq. (13) reveals that the marginal rate of substitution for consumption is the same for households holding either (real) safe assets D_t or (real) government debt B_t because both R_t and $R_t^H \mathrm{E}_t\left(1-\delta_{t+1}\right)$ equal the marginal rate of substitution. In other words, the consumption schedule remains the same whether households hold safe assets D_t or government debt B_t.

Let us define $\hat{r}_t^S \equiv \hat{r}_t^H - \hat{r}_t$, which is the interest rate spread gap for holding government debt, namely risky assets. Thus, Eq. (13) can be rewritten as:

$$\hat{r}_t^S = \delta \mathrm{E}_t\left(\hat{\delta}_{t+1}\right), \tag{14}$$

where $\sigma_B \equiv \frac{B}{Y}$ denotes the steady-state share of government debt to output. Eq. (14) reveals that the higher the expected default rate, the higher the interest rate spread, and vice versa.

Log-linearizing the definition of government debt yield R_t^H, we arrive at:

$$\hat{r}_t^S = -\frac{\phi\left(\tau + \gamma\varsigma_\tau\sigma_B\right)}{\tau + \phi\varsigma_\tau\sigma_B} sp_t + \frac{\phi\varsigma_\tau\sigma_B}{\tau + \phi\varsigma_\tau\sigma_B} b_t, \tag{15}$$

where $\phi \equiv \Gamma'(0)$ denotes the interest rate spread in the steady state and $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one-percent change in the fiscal deficit in the steady state. Following Benigno[2], we define the interest rate spread for government debt ϕ and assume $\Gamma(0) = 1$. The elasticity γ is an unfamiliar parameter, and we assume $\mid \Gamma'(\cdot) \mid < \mid \Gamma''(\cdot) \mid$; thus, $\gamma > 1$. Our assumption implies that a decrease in the fiscal surplus increases the government debt coupon rate via an increase in the interest rate multiplier, and vice versa. It also implies that changes in the government debt coupon rate are larger than the changes in the fiscal surplus in absolute value.[8]

[8] Our assumption $\gamma > 1$ is supported by the data. See Okano and Inagaki[17] for details.

The first term on the RHS of Eq. (15) displays a negative sign, implying that an increase in the fiscal surplus decreases the government debt yield, and vice versa. This is intuitively consistent because an increase in fiscal surplus decreases the interest rate multiplier. The second term on the RHS of Eq. (15) displays a positive sign, revealing that the government debt yield is an increasing function of government debt. An increase in government debt coincides with a decrease in the fiscal surplus, and vice versa. Thus, the positive sign in the second term is consistent with the negative sign in the first term.

2.2.2 Fiscal Theory of Sovereign Risk

The appropriate transversality condition for government debt is given by:

$$\lim_{j\to\infty} \beta^{t+j+1} \mathrm{E}_t \left[R^G_{t+j} \left(1-\delta_{t+j+1}\right) \frac{P_{t+j}B_{t+j}}{P_{t+j+1}} \right] = 0.$$

By iterating the second equality in Eq. (3) forward, plugging Eq. (10) into this iterated equality, and imposing the appropriate transversality condition for government debt, we arrive at:

$$C_t^{-1} R^G_{t-1} B_{t-1} \Pi_t^{-1} \left(1-\delta_t\right) = C_t^{-1} SP_t + \beta \frac{R^H_t}{R^G_t} \mathrm{E}_t \left(C^{-1}_{t+1} SP_{t+1} \right) + \beta^2 \mathrm{E}_t \left(\frac{R^H_t}{R^G_t} \frac{R^H_{t+1}}{R^G_{t+1}} C^{-1}_{t+2} SP_{t+2} \right) + \cdots, \tag{16}$$

which roughly reveals that the burden of government debt redemption with interest payment in terms of consumption, or the left-hand side (LHS), corresponds to the expected sum of the discounted value of the fiscal surplus in terms of consumption, or the RHS, because of the transversality condition. Here, $\frac{R^H_t}{R^G_t}$ and so forth appear on the RHS. An increase in the government debt coupon rate R^G_t then worsens the fiscal situation by increasing the interest payment. Thus, R^G_t is the denominator. Increased government debt yield facilitates the purchase of government debt despite decreased consumption. Decreased consumption then improves the fiscal situation because it increases the fiscal surplus in terms of consumption. Thus, R^H_t is the numerator.

Eq. (16) can be rewritten as:

$$\delta_t = 1 - \frac{\left(R^R_{t-1}\right)^{-1} \sum_{k=0}^{\infty} \prod_{h=0}^{k} \beta^k \mathrm{E}_t \left(R^R_{t+h-1} C^{-1}_{t+k} SP_{t+k} \right)}{C_t^{-1} R^G_{t-1} B_{t-1} \Pi_t^{-1}}, \tag{17}$$

where $R^R_t \equiv \frac{R^H_t}{R^G_t}$ denotes the (gross) premium difference between the government debt yield and its coupon rate. Eq. (17) is our FTSR and implies that inflation does not necessarily increase even if the government becomes insolvent, and vice versa, similar to Uribe[19]. Not only inflation but also default can mitigate the burden of government debt redemption with interest payment. In a scenario where the price level is constant and there is no inflation, if the net present value of the fiscal surplus in terms of consumption (the numerator) is about to fall below the burden of government debt redemption with interest payment in terms of consumption (the denominator), the second term on the RHS is less than unity. Simultaneously, the LHS exceeds zero; that is, default occurs. In other words, if the government becomes insolvent while the price level is strictly stable, default is inevitable. Uribe[19] revealed the trade-off between stabilizing inflation and suppressing default (hereafter the SI–SD trade-off) by introducing default, namely default risk, into the central equation of the FTPL. Similar to Uribe[19], at first glance, Eq. (17) also implies that an SI–SD trade-off. Furthermore, he calibrates and compares his model with the monetary policy rule that stabilizes inflation with the interest rate peg, under which the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration reveals that default ceases just one period after the shock decreasing the fiscal surplus, despite continuing under a monetary policy rule after the shock. This implies that a monetary policy rule to stabilize inflation includes the unwelcome possibility of magnifying default risk, which calls for an interest rate peg to counter default. Although Uribe[19] ignored the welfare perspective of these actions, his policy implications are persuasive. Paying attention only to Eq. (17), which is similar to that in Uribe's[19] model, we seemingly obtain policy implications similar to those in Uribe[19].

We now present the relationship between our FTSR, namely Eq. (17), and the FTPL. If there is neither default risk nor an interest rate multiplier in Eq. (17), Eq. (17) reduces to the following because $R_t^G = R_t^H = R_t$:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}}, \tag{18}$$

which is our version of the FTPL. On the RHS, the numerator is the net present value of the sum of the fiscal surplus in terms of consumption, and the denominator is the burden of the government debt redemption with interest payment in terms of consumption divided by inflation. The LHS represents unity. If solvency worsens, the price level increases; that is, inflation occurs, such that the burden of government debt redemption is mitigated. For now, we introduce default risk, which is no longer fully applicable, as Eq. (17) implies.

2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq. (17) one period and plugging this into Eq. (17) itself, we can rewrite Eq. (17) as a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R_{t-1}^G \Pi_t^{-1} B_{t-1}} \left\{ SP_t + \beta \mathrm{E}_t \left[\left(\frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \right) R_t^H \left(1 - \delta_{t+1} \right) B_t \right] \right\}. \tag{19}$$

In Eq. (19), the current government debt B_t appears in the second term on the RHS with a negative sign. That is, a decrease in current government debt increases the default rate, and vice versa. The sign of government debt B_t in the second term on the RHS is negative because of the transversality condition for government debt, making Eq. (17) and its second-order differential version Eq. (19) strictly applicable. That is, once issued, government debt must be redeemed. Otherwise, the burden of redemption is mitigated by default or inflation. To retain Eq. (17), once government debt is issued, the fiscal surplus must be improved as the newly issued government debt is about to reduce the fiscal surplus. Because the fiscal surplus must improve to redeem debt, the default rate declines due to an improvement in the fiscal surplus when government debt increases. Thus, the sign is negative.

Log-linearizing Eq. (19) yields:

$$\begin{aligned} c_t \quad = \quad & \mathrm{E}_t \left(c_{t+1} \right) - \hat{r}_t^H + \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} \right) + \mathrm{E}_t \left(\pi_{t+1} \right) - b_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \hat{r}_{t-1}^G - \frac{\phi \varsigma_\tau \sigma_B}{\beta \left(\tau + \phi \varsigma_\tau \sigma_B \right)} \hat{\delta}_t \\ & - \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \pi_t + \frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} b_{t-1} - \frac{\tau}{\varsigma_\tau \sigma_B} sp_t, \end{aligned} \tag{20}$$

which is our log-linearized Euler equation.

2.3 Firms

This subsection outlines the production, price setting, marginal cost, and features of the firms, which are quite similar to Gali and Monacelli[13], although here the tax is levied on firm sales and is not constant.[9]

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where A_t denotes productivity.

By combining the production function and the optimal allocation for goods $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$, we arrive at an aggregate production function related to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \tag{21}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes price dispersion.

Log-linearizing Eq. (21) yields:

$$n_t = y_t - a_t. \tag{22}$$

We assume that productivity follows an AR(1) process, namely $\mathrm{E}_t(a_{t+1}) = \rho_A a_t$, similar to government expenditure. Z_t disappears in Eq. (17) because of $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods and sets its prices $P_t(i)$ taking as given P_t and C_t. We assume that firms set staggered, Calvo pricing, according to which each seller has the opportunity to change the price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)}{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}\right)}, \tag{23}$$

where $MC_t \equiv \frac{W_t}{(1-\tau_t) P_t A_t}$ denotes the real marginal cost, $\tilde{Y}_{t+k} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ denotes the demand for goods when firms choose a new price, and $\tilde{P}_t$ denotes the newly set prices. We assume that the government levies a tax on firm sales.

By log-linearizing Eq. (23), we arrive at:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \kappa mc_t, \tag{24}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slope of the New Keynesian Phillips Curve (NKPC). Eq. (24) is the fundamental equality of our NKPC.

Substituting Eq. (11) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^{\psi}}{(1-\tau_t) A_t}. \tag{25}$$

[9] Unlike our setting, Gali and Monacelli[13] assumed that under constant employment subsidies, monopolistic power completely disappears.

It is noteworthy that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate $1-\tau$ is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it cannot be completely absorbed through taxation. Thus, we need to derive our welfare criteria following Benigno and Woodford[5] because monopolistic power is no longer removed completely, distorting the steady state.

Log-linearizing Eq. (25) yields:

$$mc_t = c_t + \psi n_t + \frac{\tau}{1-\tau}\hat{\tau}_t - a_t. \tag{26}$$

2.4 Equilibrium

The market-clearing condition requires:

$$Y_t(i) = C_t(i) + G_t(i),$$

for all $i \in [0,1]$ and all t. By plugging the optimal allocation for generic goods including Eq. (8) into this market-clearing condition, we arrive at:

$$Y_t = C_t + G_t. \tag{27}$$

By log-linearizing Eq. (27), we obtain:

$$y_t = \sigma_C c_t + \sigma_G g_t, \tag{28}$$

where $\sigma_C \equiv 1-\sigma_G$ denotes the steady-state ratio of consumption to output.

3 Welfare Costs and the LQ Problem

3.1 Derivation of the Welfare Cost Function

Following Gali[12], the second-order approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t-U}{U_C C}\right) &= \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) - \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{(1-\Phi)(1+\psi)}{\sigma_C 2}y_t^2 - \frac{(1-\Phi)(1+\psi)}{\sigma_C}y_t a_t\right.\\ &\quad \left.+\frac{\Lambda_\pi}{2}\pi_t^2\right] + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned} \tag{29}$$

with $\Lambda_\pi \equiv \frac{(1-\Phi)\varepsilon}{\sigma_C \kappa}$ where t.i.p. denotes the terms independent of policy, $o\left(\|\xi\|^3\right)$ denotes the terms of order three or higher, and $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. On the RHS, a linear term exists $\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right)$ generating the welfare reversal, which must be eliminated.

To eliminate this linear term, we follow Benigno and Woodford[4] and Benigno and Woodford[5]. By using the second-order approximated AS equation Eq. (23), the second-order approximated definition of the fiscal surplus SP_t, the second-order approximated definition of the premium difference R_t^R, the second-order approximated market clearing condition Eq. (27), and the second-order approximated government solvency condition Eq. (17), we arrive at:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{\sigma_C}y_t\right) = -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\tilde{\Omega}_1 y_t^2 - 2y_t\left(\Omega_2 g_t + \tilde{\Omega}_3 a_t\right) + \frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2\right] + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where $\hat{r}_t^R \equiv \frac{dR_t^R}{R^R}$ denotes the premium difference, $\Upsilon_0 \equiv \Theta_1 (1-\beta-\delta)^{-1} \hat{\mathcal{W}}_0 + \Theta_2 \kappa^{-1} \mathcal{V}_0$ denotes the transitory component, $\mathcal{V}_0$ denotes the initial value of the second-order approximated AS equation, $\mathcal{W}_0$ denotes that of the second-order approximated government solvency condition, and $\tilde{\Omega}_1$, Ω_2, $\tilde{\Omega}_3$, Ω_4 representing complicated building blocks of parameters with $\hat{\mathcal{W}}_t \equiv \frac{\mathcal{W}_t - \mathcal{W}_{t-1}}{\mathcal{W}_{t-1}}$, $\Lambda_r \equiv \frac{\Theta_1 \beta \left[1+(\gamma-1)^2\right]}{(1-\beta-\delta)(\gamma-1)^2}$, $\Theta_1 \equiv \frac{(1-\beta-\delta)\tau\Phi}{\Xi_0}$, $\Theta_2 \equiv -\frac{\Phi(1+2\omega_g)\omega_\phi(1-\tau)}{\Xi_0}$, $\omega_\phi \equiv 1-\beta(1-\phi) - \frac{\phi\varsigma_\tau\sigma_B}{\tau+\phi\varsigma_\tau\sigma_B}$, and Ξ_0 being building blocks.

Plugging the previous expression into Eq. (29) yields:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\frac{U_t - U}{U_C C} \right) = \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t \right) + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3 \right),$$

which is a second-order approximated utility function. The linear term is appropriately eliminated, where

$$L_t \equiv \frac{\Lambda_y}{2} \left(y_t - y_t^* \right)^2 + \frac{\Lambda_\pi}{2} \pi_t^2 + \frac{\Lambda_r}{2} \left(\hat{r}_t^R \right)^2 \tag{30}$$

denotes the period welfare cost function, and

$$y_t^* \equiv \frac{\Omega_2}{\Omega_1} g_t + \frac{\Omega_3}{\Omega_1} a_t \tag{31}$$

denotes the efficient level of output with $\Lambda_y \equiv 2\Omega_1$, $\Omega_1 \equiv \tilde{\Omega}_1 + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$.

To clarify the distinctive features of the period welfare cost function Eq. (30), we derive the period welfare cost function by using the second-order approximated government solvency condition, derived from Eq. (18). Recall that Eq. (18) is our version of the FTPL and can be derived from the government solvency condition Eq. (17) by assuming that neither default risk nor an interest rate multiplier exists. The period welfare cost function, derived by using the second-order approximated government solvency condition, which is derived from Eq. (18), is given by:

$$L_t^f = \frac{\Lambda_y^f}{2} \left(y_t - y_t^f \right)^2 + \frac{\Lambda_\pi}{2} \pi_t^2. \tag{32}$$

This is analogous to the period welfare cost function derived by Benigno and Woodford[4] where $y_t^f \equiv \frac{\Omega_2^f}{\Omega_1^f} g_t + \frac{\Omega_3^f}{\Omega_1^f} a_t$ denotes the efficient level of output when there is no default risk with $\Lambda_y^f \equiv 2\Omega_1^f$, $\Omega_1^f \equiv \tilde{\Omega}_1^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$, $\Omega_3^f \equiv \tilde{\Omega}_3^f + \frac{(1+\psi)(1-\Phi)}{2\sigma_C}$, and Ω_2^f representing a complicated block of parameters. There is no quadratic term of $\hat{r}_t^R$. Λf_y replaces Λ_y and the target level of output is not y_t^* but y_t^f in Eq. (30).

3.2 Welfare Costs in an Economy with Sovereign Risk

The most notable feature of welfare costs in an economy with default risk, namely Eq. (30), is the quadratic term of the premium difference between the government debt yield and coupon rate $\hat{r}_t^R$, the third term on the RHS. The appearance of this term suggests an opportunity cost of holding government debt. When households hold government debt, they can obtain interest income that is not necessarily at the government debt coupon rate r_t^G but may be at the government debt yield rate r_t^H. To choose the optimal consumption schedule Eq. (12), households have to maneuver their government debt position B_t, satisfying the definition of the (gross) government debt yield as follows:

$$\begin{aligned} R_t^H &= R_t \left\{ \Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right) SP^{-1} \right\} \\ &= R_t^G + R_t B_t \Gamma'\left(-sp_t\right) SP^{-1}. \end{aligned}$$

As depicted in the previous expression, the government debt coupon rate does not necessarily correspond to the government debt yield; households have to abandon income at the government debt coupon rate but can obtain income at the government debt yield on their own government debt. Thus, there is an opportunity cost to maneuvering the government debt position. Without maneuvering, households can no longer choose an optimal consumption schedule, resulting in welfare costs. This is the reason for a quadratic of the premium difference between the government debt yield and debt coupon rate $\hat{r}_t^R$ in Eq. (30).

The appearance of the quadratic of the premium difference between the government debt yield and coupon rate in Eq. (30) depends on default risk. The third term on the RHS in Eq. (30) can be rewritten as:

$$\Lambda_r \left(r_t^R\right)^2 = \Lambda_\delta \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^*\right)^2,$$

because

$$\hat{r}_t^R = \frac{\phi \varsigma_\tau \sigma_B}{\tau} \mathrm{E}_t \left(\hat{\delta}_{t+1} - \hat{\delta}_{t+1}^*\right), \tag{33}$$

which is derived by using Eqs. (5) and (14) with $\Lambda_\delta \equiv \Lambda_r \phi^2 \left(\frac{\varsigma_\tau \sigma_B}{\tau}\right)^2$ where $\hat{\delta}_t^* \equiv -\frac{\tau}{\varsigma_\tau \varsigma_B} sp_{t-1}$ denotes the target level of the default rate. The previous expression depicts the welfare costs stemming from the third term of Eq. (30) to be those of the deviation of the expected default gap from its target level $\mathrm{E}_t\left(\hat{\delta}_{t+1}^*\right)$ corresponding to the (percentage deviation of) fiscal deficit from its steady-state value $-sp_t$. In addition, the previous expression reveals that the higher the steady-state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ. Then, we have to focus on the steady-state value of interest spread ϕ, which determines that of the default rate because

$$\delta = \frac{\phi \varsigma_\tau \sigma_B}{\tau + \phi \varsigma_\tau \sigma_B}.$$

That is, the higher the steady-state value of the interest spread, the higher the steady-state value of the default, and vice versa. Because of this, the higher the steady-state value of the interest spread ϕ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ and the higher the steady-state value of the default rate δ, the higher the weights on the deviation of the expected default gap from its target level Λ_δ. In addition, when there is no interest spread in the steady state, meaning that the steady-state value of the default rate is zero, $\Lambda_\delta = 0$ is applied and the third term in Eq. (30) $\frac{\Lambda_r}{2}\left(\hat{r}_t^R\right)^2$ disappears.

Other distinctive features of Eq. (30) compared to Eq. (32) are that the weights on the output deviation from its target level Λ_y replace Λ_f and the target level output y_t^* replaces y_t^f. While Λ_y^f in Eq. (32) does not depend on the steady-state value of the interest spread ϕ, Λ_y depends on ϕ. When $\phi = 0$, $\Lambda_y = \Lambda_y^f$ is applied. That is, the difference of the weights on the output deviation from its target level depends on the steady-state value of the interest spread. In Eq. (30), the target level output y_t^* replaces y_t^f and y_t^* depends on ϕ, although y_t^f does not. When $\phi = 0$,

$y_t^* = y_t^f$. Thus, when $\phi = 0$ in which the steady-state value of the default rate is zero, namely $\delta = 0$, Eq. (32) simplifies to Eq. (30), that is,

$$L_t^* = L_t^f$$

is applied. With no default risk, the welfare cost function is analogous to one derived by Benigno and Woodford[4] who do not assume the default risk. Thus, default risk can be said to change the form of the welfare cost function.

3.3 The LQ Problem

The policy authorities minimize Eq. (30) or Eq. (32) for all t subject to Eqs. (1), (4), (5), (14), (15), (20), (22), (24), (26), and (28) and select the sequence $\left\{y_t, \pi_t, \hat{r}_t^G, \hat{r}_t, \hat{r}_t^H, c_t, b_t, mc_t, n_t, sp_t, \hat{\tau}_t, \hat{\delta}_t\right\}_{t=0}^{\infty}$. We designate the policy minimizing Eq. (30) as the '*exact*' policy because default risk exists, which policy authorities recognize. We designate the policy minimizing Eq. (32) as the '*false*' policy because although default risk exists, policy authorities do not recognize it. Eqs. (30) and (32) are not only distinguished by the quadratic term of the premium difference $\hat{r}_t^R$ but also by the weights on the output deviation from its target level and the target level output. Comparing the outcome of policy minimizing Eq. (30) with minimizing Eq. (30) without the quadratic term of the premium difference, we cannot analyze how default risk affects the outcome of the optimal policy. We have to consider not only the quadratic term of the premium difference but also the weights on the output deviation from its target level and the target level output. Thus, we compare the '*exact*' policy, which minimizes Eq. (30) with the '*false*' policy, which minimizes Eq. (32).

Under the *exact* policy, the policy authorities minimize Eq. (30), while under the *false* policy, they minimize Eq. (32). In the following, we introduce some FONCs that are worth discussing.

The FONCs for the output are given by:

$$\Lambda_y y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y y_t^*, \quad (34)$$

$$\Lambda_y^f y_t = \varsigma_\tau \rho_{7,t} + \rho_{8,t} - \rho_{10,t} + \Lambda_y^f y_t^f, \quad (35)$$

where $\rho_{7,t}$, $\rho_{8,t}$, and $\rho_{10,t}$ are Lagrange multipliers on Eqs. (1), (22), and (28), respectively. Eq. (34) is the FONC under the *exact* policy, revealing that the target level output is the most efficient level of output. Eq. (35) is the FONC under the *false* policy, revealing that the target level output is the most efficient level of output when there is no default risk.[10]

The FONC for inflation is given by:

$$\Lambda_\pi \pi_t = -\frac{\tau + \varsigma_\tau \sigma_B}{\varsigma_\tau \sigma_B} \rho_{1,t} + \frac{1}{\beta} \rho_{1,t-1} - (\rho_{2,t} - \rho_{2,t-1}) - \frac{\tau}{\beta (\tau + \phi \varsigma_\tau \sigma_B)} \rho_{6,t}, \quad (36)$$

where $\rho_{1,t}$ and $\rho_{6,t}$ are Lagrange multipliers on Eqs. (20) and (4), respectively. Because of commitment, lagged Lagrange multipliers appear in Eq. (36). Eq. (36) is common to both *exact* and *false* policies. In Eqs. (34), (35), and (36), $\rho_{2,t}$ appears and those equalities imply that inflation is stabilized by stabilizing output (or stabilizing the welfare-relevant output gap or the difference between output and the target level output). This mechanism is similar to others in the literature on optimal monetary policy and common to both the *exact* and *false* policies.

[10]To derive Eqs. (34) and (35), we use the FONCs for the marginal cost and the hours of labor, eliminating Lagrange multipliers on Eq. (22) $\rho_{8,t}$.

The FONCs for the government coupon gap are given by:

$$\Lambda_r \hat{r}_t^G = \Lambda_r \hat{r}_t^H - \rho_{5,t} + \frac{\beta (\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \quad (37)$$

$$0 = -\rho_{5,t} + \frac{\beta (\tau + \varsigma_\tau \sigma_B)}{\varsigma_\tau \sigma_B} \rho_{1,t+1} + \frac{\tau}{\varsigma_\tau \sigma_B} \rho_{6,t}, \quad (38)$$

where $\rho_{5,t}$ is a Lagrange multiplier on Eq. (3) and Eqs. (37) and (38) are the FONCs under the *exact* and *false* policies, respectively. Eq. (37) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq. (38) implies no explicit incentive to minimize it.

The FONCs for the government debt yield gap are given by:

$$\Lambda_r \hat{r}_t^H = \Lambda_r \hat{r}_t^G - \rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \quad (39)$$

$$0 = -\rho_{1,t} + \rho_{3,t} - \rho_{4,t}, \quad (40)$$

where $\rho_{4,t}$. Eqs. (39) and (42) are the FONCs under the *exact* and *false* policies, respectively. Eq. (39) implies that the policy authorities have to minimize the premium difference $\hat{r}_t^R \equiv \hat{r}_t^H - \hat{r}_t^G$ to minimize the welfare costs, although Eq. (42) implies no explicit incentive to minimize it.

4 Numerical Analysis

4.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. The calibrated parameters are depicted in Table 1.[11] In addition, we assume that productivity a_t and government expenditure g_t follow AR(1) processes and that persistence is 0.9.

4.2 Impulse Response Functions

We next discuss the IRFs. Figs. 1 and 2 depict the IRFs to a unit decrease in productivity and to a unit increase in government expenditure, respectively. First, we discuss IRFs of a one-percent decrease in productivity (Fig. 1). A decrease in productivity decreases the target level output under both policies as demonstrated in the definition of the target level output. Although the target level output depends on the steady-state value of the interest spread under the *exact* policy, the difference in the target level output can be ignored (Panel 3). Under the *false* policy, the tax gap falls enough to boost output although it does not under the *exact* policy (Panel 9). As a result, output and the welfare-relevant output gap under the *exact* policy fall more under the *exact* policy, while under the *false* policy, output corresponds approximately to its target level and the welfare-relevant output gap is close to zero (Panels 1 and 2). By stabilizing the welfare-relevant output gap, inflation is stabilized under the *false* policy but not under the *exact* policy (Panel 4). This is why we have a prescription for implementing monetary and fiscal policy that 1) policy authorities should not suppress inflation aggressively if the causation of default crisis is rooted by a decrease in productivity (see Section 1).

On the one hand, a drastic decrease in the tax gap worsens the fiscal surplus under the *false* policy; on the other hand, under the *exact* policy, it does not because the tax gap is not lowered

[11] Creedy and Gremmell[?] report that the tax revenue elasticity ranges from 0.5 to 1; we choose 1 as the tax revenue elasticity ς_τ.

very much. Thus, government solvency worsens under the *false* policy and the default gap increases and continues although government solvency is healthier and the default gap is less under the *exact* policy (Panels 6 and 7). Here, the fiscal surplus under the *exact* policy plays an important role in stabilizing the premium difference, which is:

$$
\begin{aligned}
\hat{r}_t^R &\equiv \hat{r}_t^H - \hat{r}_t^G \\
&= \hat{r}_t^H - \hat{r}_t - \left(\hat{r}_t^G - \hat{r}_t\right) \\
&= \hat{r}_t^S + \phi sp_t \\
&= -\frac{\phi\varsigma_\tau\sigma_B\left(\gamma-\phi\right)}{\tau+\phi\varsigma_\tau\sigma_B} sp_t + \frac{\phi\varsigma_\tau\sigma_B}{\tau+\phi\varsigma_\tau\sigma_B} b_t \\
&= \frac{\phi\varsigma_\tau\sigma_B}{\tau}\mathrm{E}_t\left(\hat{\delta}_{t+1} - \delta_{t+1}^*\right).
\end{aligned}
$$

Substituting Eq. (15) into the third line, we reveal that an increase in the fiscal surplus and a decrease in the government debt decreases the premium difference, as long as the steady-state value of the interest rate spread is not high (See line 4 in the previous expression). Because inflation is not so stabilized, government debt decreases more under the *exact* policy, as depicted in Eq. (4) (Panel 10). Because of a small decrease in the tax gap, the effect of a decrease in the fiscal surplus is not severe under the *exact* policy (Panels 7 and 9). Thus, the premium difference is more aggressively stabilized under the *exact* policy although it rises considerably under the *false* policy (Panel 5). As shown in line 5 in the previous expression, the smaller the premium difference, the smaller the deviation of the expected default rate from its target level. Reflecting this fact, the default gap is well stabilized under the *exact* policy although it does not converge immediately under the *false* policy (Panel 6).

The default gap under the *exact* policy, which rises sharply after the shock, is consistent with Uribe[19]'s result. Uribe analyzed the "interest rate peg" monetary policy that pegs the nominal interest rate for risky assets to that for safe assets under an economy with default risk. His interest peg policy raises the default rate sharply after an exogenous negative fiscal surplus shock although a rise in the default rate is stabilized immediately. His policy corresponds to the that minimizing the interest spread for risky assets $\hat{r}_t^S$ in our paper. As Eq. (14) implies, the policy minimizing the interest spread for risky assets is equivalent to the policy minimizing the expected default gap $\mathrm{E}_t\left(\hat{\delta}_{t+1}\right)$. If the policy authorities successfully adopt the policy minimizing the interest spread for risky assets, the expected default rate becomes zero. Thus, although a rise in the default gap immediately after the shock is inevitable, the default gap then becomes zero. Our *exact* policy has a feature minimizing the default gap itself, as the previous expression implies, and the default gap rises sharply after the shock and converges immediately, although full convergence takes time. This explains why our result is consistent with Uribe[19].

Next, we discuss the IRFs of a one-percent increase in government expenditure (Fig. 2). An increase in government expenditure increases the target level output and applies to inflation pressure (Panel 3). To stabilize inflation, the tax gap is hiked under both *exact* and *false* policies although the fiscal surplus worsens. Consequently, inflation is well stabilized under both *exact* and *false* policies (Panel 4). This is why our prescription for implementing monetary and fiscal policy contends that 2) when an increase in government expenditure gives rise to default risk, policy authorities should stabilize inflation, similar to a situation where there is no default risk (see Section 1). Due to a decrease in the fiscal surplus, the premium difference increases under both policies (Panel 5). The fluctuation of the default gap under both policies is almost the same and not severe even under the *false* policy (Panel 6). The reason it is not severe is that the interest spread for risky assets r_t^S does not differ much between the *exact* and *false* policies. As depicted in Eq. (15), the interest spread for risky assets depends on the government debt and its fluctuation under the *false* policy is close to that under the *exact* policy (Panel 10). Thus, the premium difference $\hat{r}_t^R$ under the *false* policy is not severe (recall that the premium difference consists of the coupon premium $\hat{r}_t^G - \hat{r}_t$ and the interest spread for the risky assets r_t^S).

In other words, whether conducting the *false* or *exact* policies, an increase in the tax gap is necessary to stabilize inflation, to an increase in the government expenditure. This increase in the tax gap applies pressure to improve government solvency. A decrease in the fiscal surplus under the *false* policy is very close to the *exact* policy and less than it in the case of a decrease in productivity (Panel 7). Because a worsening in the government solvency is not severe, the default gap under the *false* policy is very close to the *exact* policy and less than it to a decrease in productivity and the premium difference $\hat{r}_t^R$ under the *false* policy is not severe, as mentioned.

For reference, we note that the standard deviation of the default gap to a one-percent increase in government expenditure under the *exact* policy is 1.3839 and under the *false* policy, it is 1.3684 (6th row, Table 2). These are almost same. On the one hand, the standard deviation of the default gap to a one-percent decrease in productivity under the *exact* policy is 3.7691 and under the *false* policy it 8.6016 (6th row, Table 2). These are different and are higher than it to an increase in government expenditure.

4.3 Effects of Differences of Policy on Productivity and Government Expenditure Shocks

Comparing Fig. 1 with Fig. 2 reveals nonnegligible differences between policies on productivity and government expenditure shocks. As depicted in Panel 9 in Fig. 1, the tax gap is severely reduced under the *false* policy, but only slightly reduced under the *exact* policy. In addition, while the nominal interest gap is hiked under the *false* policy, it does not fluctuate under the *exact* policy, as shown in Panel 8 in Figs. 1 and 2. To cope with the productivity shock, policy instruments are manipulated inversely. However, as depicted in Panels 8 and 9 in Fig. 2, both the tax gap and nominal interest rates are similarly manipulated. The nominal interest rate gap decreases and the tax gap increases. Differences are almost zero in response to government expenditure shock between *exact* and *false* policies.

The reason response variation of monetary and fiscal policies depends on the type of shock relates to how each shock shifts the NKPC. Plugging Eqs. (22), (26), and (28) and the definition of the efficient level of output into Eq. (24), we arrive at:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \frac{\kappa(1+\sigma_C\psi)}{\sigma_C}\hat{y}_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t + \frac{\kappa(1+\sigma_C\psi)\Omega_2}{\sigma_C\Omega_1}g_t - \frac{\kappa[(1+\psi)\sigma_C\Omega_1 - (1+\sigma_C\psi)\Omega_3]}{\sigma_C\Omega_1}a_t, \quad (41)$$

where $\hat{y}_t \equiv y_t - y_t^*$ denotes the welfare-relevant output gap. In the previous equality, the coefficients of the fourth and fifth terms are direct effects that shift the NKPC through changes in government expenditure and productivity, respectively. Because of the distorted steady state, both government expenditure and productivity appear on the NKPC, unlike in the model assuming that monopolistic competitive power is completely dissolved. Under our benchmark parameterization, the coefficient of government expenditure is 0.0144, although that of productivity is 0.1877. While the direct effects of shifting the NKPC stemming from an increase in government expenditure are negligible, those stemming from a decrease in productivity are nonnegligible; in other words, that decrease (increase) changes inflation strongly.

The reason productivity shifts the NKPC strongly stems from a distorted steady state. In our model, tax is levied on output and monopolistic competitive market power remains as long as the tax rate is set to make the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor Φ. To attain $\Phi = 0$, under our setting with tax levied on output, the tax rate should be negative as long as the elasticity of substitution among goods is larger than one, namely $\varepsilon > 1$. Under our benchmark parameterization, the steady-state value of the tax rate should be –0.2, which is unrealistic. Because of a positive tax rate in the steady state, the productivity shock is seen as a cost–push shock. The higher the steady-state value of the tax rate, the lower the contribution of changes in productivity to increase the target level output y_t^*. The complicated parameter Ω_3, a function of the steady-state value of τ, appears in the definition of the target level of output Eq. (31) and the NKPC Eq. (43). As mentioned, the higher the steady-state value of the tax rate, the lower Ω_3. A decrease in productivity decreases the target level output. However, if the steady-state value of the tax rate is larger than its optimal value, i.e., –0.2, the target level output does not decrease enough. Thus, the output does not decrease enough. In contrast, a decrease in productivity applies pressure to increase marginal cost, as shown in Eq. (26). Now, Ω_3 is small and a decrease in the target level output cannot absorb the decrease in productivity. This can be explained by the coefficient of productivity in Eq. (43) as follows:

$$\kappa\left(1+\psi\right) - \frac{\kappa\left(1+\sigma_C\psi\right)\Omega_3}{\sigma_C\Omega_1}.$$

In this expression, the first term is the direct effect that increases inflation stemming from a decrease in productivity and the second term is the pressure to decrease inflation through a decrease in the target level output. Because the steady-state value of the tax rate exceeds its optimal value, the direct effect to increase inflation exceeds the pressure to decrease the target level output. Thus, productivity works as a cost–push shock.

In contrast, the shift of NKPC through an increase in government expenditure is negligible, due to a “non-Keynesian effect”. Although government expenditure increases output, consumption decreases simultaneously, as depicted in Panel 11 in Fig. 2. The pressure to increase the marginal cost is canceled by the decrease in consumption, and the upward shift of NKPC through an increase in government expenditure is minimal or negligible.

Under the *exact* and the *false* policies, policy authorities have considerably different period loss functions if the steady-state value of the interest spread is high. While policy authorities under the *exact* policy do not necessarily focus only on stabilizing inflation, under the *false* policy, they almost solely focus on stabilizing inflation. However, an increase in government expenditure does not shift the NKPC much, and inflation stabilization policy does not necessarily worsen welfare costs even under the *false* policy. Thus, as long as government expenditure affects the economy, the policy response does not differ much between the two policies. This implies that even if default risk exists, policy authorities are not necessarily aware of this risk. However, if productivity changes are observed, policy authorities should become aware of the risk. Strong pressure exists to increase inflation and use the *false* policy to try to stabilize it through a decreasing tax gap, which causes high and long-lasting default because of a worsening fiscal surplus, although the *exact* policy does not cause this result. Awareness of default risk is important if productivity changes are observed.

5 Optimal Monetary and Fiscal Policy Rules and Welfare Costs

5.1 Optimal Monetary and Fiscal Policy Rules

This section introduces simple policy rules. The monetary policy rule follows a class of Taylor rules:

$$\hat{r}_t = \varphi_\pi \pi_t, \tag{42}$$

while the rule for the government conducting fiscal policy takes the form:

$$\hat{\tau}_t = \varphi_b b_{t-1}. \tag{43}$$

We find both φ_π and φ_b through grid search, which minimizes the consequent difference in welfare costs under optimal monetary and fiscal policy. That is, we find φ_π and φ_b that replicate welfare costs under optimal monetary and fiscal policy. The ranges of grid search are limited to $\varphi_\pi \in [1, 30]$ and $\varphi_b \in [0, 3]$ for the condition of determinacy. The numbers of grid are 25 for both coefficients. Lines 4 and 5 in Table3 depict φ_π and φ_b under *exact* and the *false* policy rules. As revealed in columns 2 and 5, φ_π and φ_b under the *false* policy rule are larger than under the *exact* policy. This implies that inflation is more aggressively stabilized under the *false* policy, as period loss function Eqs. (30) and (32).

For changes in productivity, although φ_π under the *exact* policy is higher than under the *false* policy, φ_b under the *exact* policy is smaller than under the *false* policy and is zero (see columns 3 and 6). This implies that the *exact* policy is not necessarily the policy that tends to stabilize inflation aggressively following changes in productivity, consistent with what Panel 4 in Fig. 1 reveals. For changes in government expenditure, φ_π under the *exact* policy is larger than it is following changes in productivity (see columns 3 and 4). In addition, φ_b under the *false* policy is the same as under the *exact* policy (see columns 4 and 7). The two facts imply that the *exact* policy is not necessarily the policy that avoids stabilizing inflation, and awareness of default risk is not as important as long as changes in government expenditure hit an economy with default risk. These implications are consistent with our discussion.

5.2 Welfare Analysis

We now analyze the welfare properties of both policies. The expected welfare costs are given by:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t \right),$$

which is the first term of a second-order approximated utility function and the sum of discounted period welfare costs. The last line in Table 3 depicts welfare costs under both the *exact* and *false* policy rules.[12] On average, welfare costs under the *exact* policy are smaller than under the *false* policy. Welfare gains from conducting the *exact* policy ((welfare costs under the *false* policy – welfare costs under the *exact* policy) / welfare costs under the *false* policy), namely awareness of default risk, are 64.4% following changes in both productivity and government expenditure. When

[12] Instead of the FONCs for policy authorities, monetary policy rule Eq. (42) and fiscal policy rule Eq. (43) are included in the model to calculate welfare costs.

productivity changes, welfare gains from conducting the *exact* policy are 49.7%. These results are consistent with our intuition. However, welfare gains from conducting the *exact* policy are only 1.0% and are thus negligible.

These results generate our policy prescriptions that 3) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is an increase in government expenditure are negligible, while 4) welfare gains from conducting optimal monetary and fiscal policy with awareness of default risk when there is a decrease in productivity are nonnegligible (see Section 1).

6 Conclusion

We developed a class of DSGE models with nominal rigidities and introduced default risk to the model. In response to a decrease in productivity, if the focus is solely on stabilization in inflation, the fiscal authority decreases the tax rate to stabilize inflation by boosting output. Although inflation is stabilized, default is inevitable because a decrease in tax rate harms government solvency. If the aim is to stabilize inflation and suppress default, that is, to implement an optimal monetary and fiscal policy, in response to a decrease in productivity, although the fiscal authority decreases tax rate, the decrease is less, partially suppressing default and stabilizing inflation. If productivity changes are observed, policy authorities should be aware of default risk.

In response to an increase in government expenditure, if the focus is solely on stabilizing inflation, the fiscal authority increases tax rate to stabilize inflation by canceling the pressure boosting output resulting from an increase in government expenditure. This increase in tax rate improves government solvency. Thus, stabilizing inflation causes suppressing default, achieving a result similar to that under optimal policy. That is, being aware of such risk is not very important following government expenditure changes.

We also found that welfare gains from awareness of default risk are nonnegligible if productivity changes, although welfare gains from awareness of default risk are minimal if government expenditure changes.

Although we are motivated by the European debt crisis, as mentioned in section 1, our closed economy model is not consistent with the Eurozone adopting a unified currency. Our closed economy setting allows policy authorities to implement monetary and fiscal policies in a coordinated manner, whereas these policies are segregated in the Eurozone. Additionally, our closed economy setting ignores how a fixed exchange rate affects dynamics. Further research is thus necessary to obtain more realistic policy prescriptions.

Appendices

A Nonstochastic Steady State

We focus on equilibria, where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$ and $\frac{\tilde{P}_t}{P_t} = 1$. Because this steady state is nonstochastic, productivity has unit values, i.e., $A = 1$.

In this steady state, the gross nominal interest rate equals the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Because $\Gamma(0) = 1$, the definition of the government debt coupon rate simplifies to:

$$R^G = R.$$

Notice that $sp_t = 0$ in the steady state.

Eq. (23) can be rewritten as:

$$\frac{\tilde{P}_t}{P_t} = \mathrm{E}_t\left(\frac{K_t}{F_t}\right), \tag{A.1}$$

with

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1}\sum_{k=0}^{\infty}\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{t+k}MC_{t+k}^n \quad ; \quad F_t \equiv P_t\sum_{k=0}^{\infty}\left(P_{t+k}C_{t+k}\right)^{-1}\tilde{Y}_{t+k},$$

which simplifies in the steady state to

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}YMC^n}{(1-\alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1-\alpha\beta)(PC)}.$$

Plugging those equalities into the steady-state condition of Eq. (A.1), namely $K = F$, yields:

$$P = \frac{\varepsilon}{\varepsilon - 1}MC^n,$$

which can be rewritten as

$$MC = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-1}. \tag{A.2}$$

Furthermore, Eqs. (25) and (A.2) imply the following:

$$\frac{U_N}{U_C} = \frac{1-\tau}{\left(\frac{\varepsilon}{\varepsilon-1}\right)\mu^w} = 1 - \Phi,$$

with $U_C = C^{-1}$ and $U_N = N^{\psi}$. Because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

The definition of R_t^H in the steady state, simplifies to

$$R^S = \left[1 + \frac{B}{SP}\Gamma'(0)\right]. \tag{A.3}$$

Eq. (13) simplifies in the steady state to

$$R^S = (1-\delta)^{-1}. \tag{A.4}$$

By plugging Eq. and rearranging (A.4) into Eq. (A.3), we arrive at:

$$\delta = \frac{\phi\varsigma_\tau\sigma_B}{\tau + \phi\varsigma_\tau\sigma_B},$$

where we use $\frac{B}{SP} = \left(\frac{SP}{Y}\right)^{-1}\frac{B}{Y}$.

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Table 1: Parameterization

Parameter	Value	Source
β	0.99	Gali and Monacelli[13]
ψ	3	Gali and Monacelli[13]
θ	0.66	Gali and Monacelli[13]
ε	6	Gali and Monacelli[13]
ϕ	0.033	Okano and Inagaki[17]
γ	1.1736	Okano and Inagaki[17]
τ	0.3	Ferrero[11]
σ_G	0.276	Ferrero[11]
σ_B	2.4	Ferrero[11]
ς_τ	1	Creedy and Gremmell[9]

Table 2: Macroeconomic Volatilities

		Exact			False	
Shocks	All	Productivity	Government Expenditure	All	Productivity	Government Expenditure
$y_t - y_t^*$	0.3956	0.3956	0.0046	0.0015	0.0010	0.0011
π_t	0.0483	0.0483	0.0010	0.0000	0.0000	0.0000
$\hat{r}_t^R$	0.6113	0.6113	0.0073	1.9594	1.9593	0.0217
$\hat{\delta}_t$	4.0151	3.7691	1.3839	8.7098	8.6016	1.3684
$\hat{r}_t$	0.2867	0.2691	0.0989	0.9488	0.9444	0.0914
$\hat{\tau}_t$	2.1719	1.4122	1.6501	5.8779	5.6481	1.6274

Table 3: Optimal Monetary and Fiscal Policy Rules and Welfare Costs

		Exact			False	
Shocks	All	Productivity	Government Expenditure	All	Productivity	Government Expenditure
φ_π	14.0556	9.8421	10.1579	23.5556	6.8947	17.7895
φ_b	0.3333	0	2.6842	3	3	2.6842
$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 (L_t)$	1.3476	1.3270	0.6275	3.7885	2.6545	0.6336

Figure 1: IRFs to Unit Decrease in Productivity

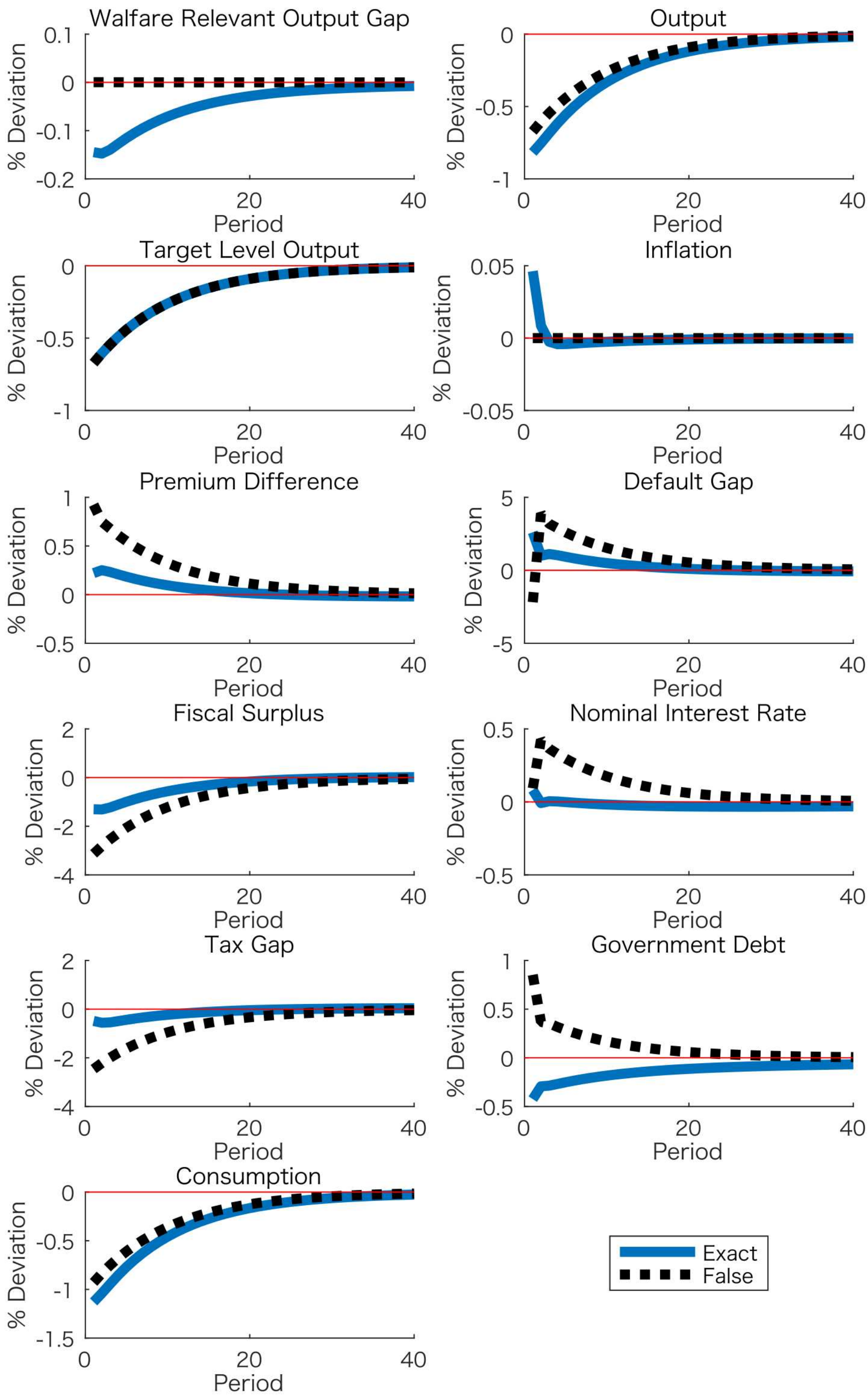

Figure 2: IRFs to Unit Increase in Government Expenditure